

Neutrality Delays Resolution: An Expected Resolution-Time Bound

A Technical Note on the S_2 Signature

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Abstract

Principle. In systems where competing outcomes resolve through evidence accumulation under constraints, weak effective bias should produce longer expected resolution times; this is the operational content of the S_2 stabilization signature. **Result.** We formalize the simplest case as a one-dimensional drift-diffusion process with fixed symmetric absorbing boundaries and start point $z = 0$: as the drift magnitude $|\mu|$ decreases toward zero, the expected first-passage time $E[\tau]$ increases strictly monotonically and attains the diffusion-only ceiling a^2/σ^2 at zero drift. The proof is self-contained from the Kolmogorov backward equation. The result yields a cross-domain latency curve testable wherever outcome resolution can be modeled as evidence accumulation against absorbing thresholds and the constraint asymmetry can be measured independently of latency. Sequential testing, metastability, and absorbing-Markov-chain pictures are noted as analogies, not as parallel bounds. The claim is conditional on the stated formalism; it is operational, not philosophical.

Review target

Acceptance criterion. Acceptance of this note commits the reader only to the following: under the stated one-dimensional drift-diffusion model with constant drift, constant noise, fixed symmetric absorbing boundaries, and start point $z = 0$, expected first-passage time is strictly maximized at zero drift and decreases monotonically as drift magnitude increases. Acceptance does not commit the reader to Universal Collapse Theory as a metaphysical framework, to the universality of S_2 across domains, or to any empirical claim unless the domain independently justifies the stated formalism and independently measures constraint asymmetry.

Rejection criterion. The note should be rejected or revised if the first-passage theorem is mathematically incorrect, if the drift-diffusion formalism is structurally inadequate for its intended use as an S_2 test template, if constraint asymmetry cannot be measured independently of latency, or if the proposed latency curve cannot distinguish neutrality-induced delay from changes in boundary placement, effective noise, starting-point bias, or hidden asymmetry.

Stack placement

This note serves a single role in the Universal Collapse Theory library: it provides a citable conditional lemma for the S_2 stabilization signature, paralleling *Objectivity from Records* (Jones, 2026d) for S_1 . Three downstream documents reference S_2 and need a formal anchor — *Records Across Nature, Life, and Mind* (Jones, 2026a) names S_2 as a portable empirical signature, *The Structuralization of Empiricism* (Jones, 2026b) uses S_2 to describe how weakly biased constraint fields extend resolution latency, and the *Update Integrity Standard* (Jones, 2026c) treats S_2 plateau duration as a candidate stopping criterion under independence-audit discipline. This note supplies the simplest first-passage bound those documents can

cite. Operational pair: Methods-S₂ (*Auditing Constraint Asymmetry in Latency-Based Resolution Tests*). Together they constitute the S₂ formal–protocol pair — this Technical Note proves the formal bound; the paired Methods Paper translates it into a deployable audit protocol. Readers not evaluating the broader UCT corpus may treat this section as orientation only; it is not load-bearing for the mathematical claim in §2.

1. Setup and definitions

Process. Let Z_t be a one-dimensional drift-diffusion process satisfying

$$dZ_t = \mu dt + \sigma dW_t, \quad Z_0 = 0,$$

with drift $\mu \in \mathbb{R}$, noise scale $\sigma > 0$, and W_t a standard Brownian motion. **Resolution boundaries.** The process resolves when Z_t reaches either of the absorbing boundaries at $\pm a$ with $a > 0$:

$$\tau = \inf \{ t \geq 0 : |Z_t| = a \}.$$

Bias. The drift μ represents the effective constraint bias toward one resolution boundary. $\mu = 0$ is full neutrality (symmetric resolution); $|\mu| > 0$ is asymmetric constraint pressure favoring one outcome.

S₂ claim. Expected resolution time $E[\tau]$ is a monotone non-increasing function of $|\mu|$, and is maximized at $\mu = 0$ with the diffusion-only value $E[\tau | \mu=0] = a^2 / \sigma^2$.

2. Theorem 1 (drift-diffusion case)

Theorem 1 (Neutrality Maximizes Expected Resolution Time). Let Z_t be the drift-diffusion process above, with $\mu \in \mathbb{R}$, $\sigma > 0$, absorbing boundaries at $\pm a$, and $Z_0 = 0$. Then:

(a) The expected first-passage time admits the closed form

$$E[\tau] = (a / \mu) \tanh(a\mu / \sigma^2) \quad \text{for } \mu \neq 0, \quad \text{and} \quad E[\tau | \mu = 0] = a^2 / \sigma^2.$$

(b) $E[\tau]$ is a strictly decreasing function of $|\mu|$, with maximum a^2 / σ^2 at $\mu = 0$.

(c) **Explicit threshold.** For any $0 < \varepsilon < a^2 / \sigma^2$, there exists a unique critical bias μ_ε such that $|\mu| \geq \mu_\varepsilon$ implies $E[\tau] \leq a^2 / \sigma^2 - \varepsilon$. Equivalently, $\mu_\varepsilon = (\sigma^2 / a) \cdot x_\varepsilon$, where x_ε is the unique positive solution of $(\tanh x) / x = 1 - \varepsilon \sigma^2 / a^2$; for small ε , $\mu_\varepsilon \approx (\sigma^3 / a^2) \cdot \sqrt{(3\varepsilon)}$.

Proof.

Statement (a). Let $u(z) = E[\tau | Z_0 = z]$ for $z \in [-a, a]$. By the standard Kolmogorov backward equation for expected exit times (see Karatzas & Shreve, 1991, §2.8; Redner, 2001, §1.5),

$$(\sigma^2/2) u''(z) + \mu u'(z) = -1, \quad u(\pm a) = 0.$$

A particular solution to the inhomogeneous equation is $u_p(z) = -z/\mu$. The homogeneous equation has roots $r = 0$ and $r = -2\mu/\sigma^2$, giving the general form $u(z) = A + B \cdot \exp(-2\mu z/\sigma^2) - z/\mu$. Imposing $u(\pm a) = 0$ and writing $\alpha = 2a\mu/\sigma^2$ yields $A = (a/\mu) \coth \alpha$ and $B = -a/(\mu \sinh \alpha)$. Evaluating at $z = 0$ and using $(\cosh \alpha - 1)/\sinh \alpha = \tanh(\alpha/2)$,

$$E[\tau] = u(0) = (a/\mu) \tanh(a\mu/\sigma^2) \quad \text{for } \mu \neq 0.$$

Letting $\mu \rightarrow 0$ in this expression recovers $E[\tau | \mu = 0] = a^2/\sigma^2$ via $\tanh(x)/x \rightarrow 1$. $\square$

Statement (b). Let $x = a\mu/\sigma^2$ and $h(x) = \tanh(x)/x$ for $x \neq 0$, $h(0) = 1$. Then $E[\tau] = (a^2/\sigma^2) \cdot h(x)$. By symmetry $h(x) = h(-x)$, so it suffices to show h is strictly decreasing on $(0, \infty)$. Differentiating,

$$h'(x) = (x \operatorname{sech}^2 x - \tanh x) / x^2,$$

so the sign of $h'(x)$ is the sign of $q(x) = x \operatorname{sech}^2 x - \tanh x$. We have $q(0) = 0$ and

$$q'(x) = -2x \operatorname{sech}^2 x \cdot \tanh x.$$

For $x > 0$, every factor on the right-hand side has unambiguous sign — $-2x < 0$, $\operatorname{sech}^2 x > 0$, $\tanh x > 0$ — so $q'(x) < 0$ on $(0, \infty)$. Hence q is strictly decreasing on $(0, \infty)$, and since $q(0) = 0$ we have $q(x) < 0$ on $(0, \infty)$, giving $h'(x) < 0$ on $(0, \infty)$. $E[\tau]$ is therefore strictly decreasing in $|\mu|$, with maximum a^2/σ^2 at $\mu = 0$. $\square$

Statement (c). Strict monotonicity of h on $[0, \infty)$, together with $h(0) = 1$ and $h(x) \rightarrow 0$ as $x \rightarrow \infty$, gives a unique $x_\varepsilon \in (0, \infty)$ with $h(x_\varepsilon) = 1 - \varepsilon\sigma^2/a^2$ for any $0 < \varepsilon < a^2/\sigma^2$. Setting $\mu_\varepsilon = (\sigma^2/a) x_\varepsilon$ then yields the explicit threshold of part (c). The small- ε asymptotic follows from the expansion $h(x) = 1 - x^2/3 + O(x^4)$, giving $x_\varepsilon \approx \sqrt{(3\varepsilon\sigma^2/a^2)}$ and $\mu_\varepsilon \approx (\sigma^3/a^2) \sqrt{(3\varepsilon)}$. $\square$

3. Remarks and extensions

Sequential-testing interpretation. Wald’s sequential probability ratio test (SPRT) gives an analogous result for discrete-time evidence accumulation: under hypotheses with low Kullback–Leibler divergence (low effective drift), the expected number of samples to commit grows roughly inversely in the divergence (Wald, 1947). The present continuous-time bound and Wald’s discrete-time bound describe the same phenomenon — neutrality slows resolution — in different formalisms.

Metastability / Kramers interpretation. For a one-dimensional gradient system $dZ_t = -V'(Z_t) dt + \sigma dW_t$ with double-well potential V , classical Kramers theory predicts that escape times grow exponentially in the barrier-height-to-noise ratio (Kramers, 1940; Hänggi, Talkner & Borkovec, 1990). When competing wells have comparable effective stability, commitment probabilities become less biased and resolution may be delayed depending on potential geometry and initial ensemble. Drift-diffusion can approximate local biased diffusion along a one-dimensional reaction coordinate in suitable regimes, but Kramers escape and Theorem 1 remain distinct metastability statements rather than parallel bounds.

Markov chain interpretation. For a finite absorbing Markov chain with transient transition submatrix Q , expected absorption times are computed from the fundamental matrix $N = (I - Q)^{-1}$ (Kemeny & Snell, 1960). Eigenvalues of Q near 1 — equivalently, small spectral gaps — produce long expected absorption times, giving a discrete-state analogue of delayed resolution under near-symmetric transitions. The exact bound depends on the transition structure; the connection to Theorem 1 is again qualitative, not a derivation.

Beyond one dimension. In higher dimensions, expected first-passage time depends on the geometry of the resolution set and on the full drift vector, not on a scalar bias. The qualitative S_2 hypothesis — that resolution time grows as effective constraint asymmetry decreases — may extend beyond this one-dimensional setting, but rigorous bounds require problem-specific analysis.

4. Empirical signature (latency curve)

Prediction. In any system where outcome resolution can be modeled as evidence accumulation against absorbing decision thresholds, expected resolution latency satisfies $E[\tau] \approx g(|\Delta K|)$ where ΔK is the audited constraint asymmetry between alternatives and g is monotone non-increasing in $|\Delta K|$ with a finite ceiling at $\Delta K = 0$ under the formalism’s symmetric-absorbing-boundary assumptions.

Protocol. Vary ΔK across conditions. Estimate or audit the effective constraint asymmetry independently of the latency measurement (avoiding circular inference). Fit $E[\tau]$ vs $|\Delta K|$ against the drift-diffusion form or a domain-appropriate analogue; report the ceiling and the slope at small $|\Delta K|$.

Identification requirements. The latency curve is interpretable as S_2 evidence only if (i) ΔK is estimated independently of latency, (ii) decision boundaries are fixed across conditions or separately estimated, (iii) effective noise is measured or modeled, (iv) starting-point bias is controlled or fitted, and (v) censoring or

timeout rules are reported. A monotone latency increase near $|\Delta K| = 0$ is insufficient by itself; the fitted model must rule out boundary expansion, reduced noise, changing initial conditions, or unmeasured task difficulty as alternative explanations. The TN- S_1 independence-audit discipline applies here as well: the constraint asymmetry that drives the latency prediction must be identified by an evidence channel separable from the latency it is asked to explain.

Failure condition. For the latency version of S_2 formalized here, the signature fails locally if independently measured constraint symmetry does not increase expected resolution latency under the specified domain model, or if the latency curve saturates without reaching a ceiling that scales with the diffusion or noise scale of the system. Broader S_2 observables, such as plateau duration or state-distribution entropy, require their own domain-specific formalization.

Domains. Primary instance where the drift-diffusion formalism is directly underwritten: perceptual decision latency under near-threshold stimuli (Ratcliff & McKoon, 2008). Closely related first-passage instances include developmental or cellular event timing under stochastic threshold-crossing models, provided the domain model independently specifies the state variable, boundary, drift, and noise terms. Candidate instances requiring a formalism extension (Kramers escape, absorbing Markov, or analogous): phenotypic switching under fluctuating selection, neutral-network traversal in genotype-phenotype maps, and institutional commitment delay under symmetric policy alternatives. The lemma underwrites the latency curve only where the modeling assumptions are independently justified for the domain in question; downstream T16 empirical demonstrations carry that justification on a per-domain basis.

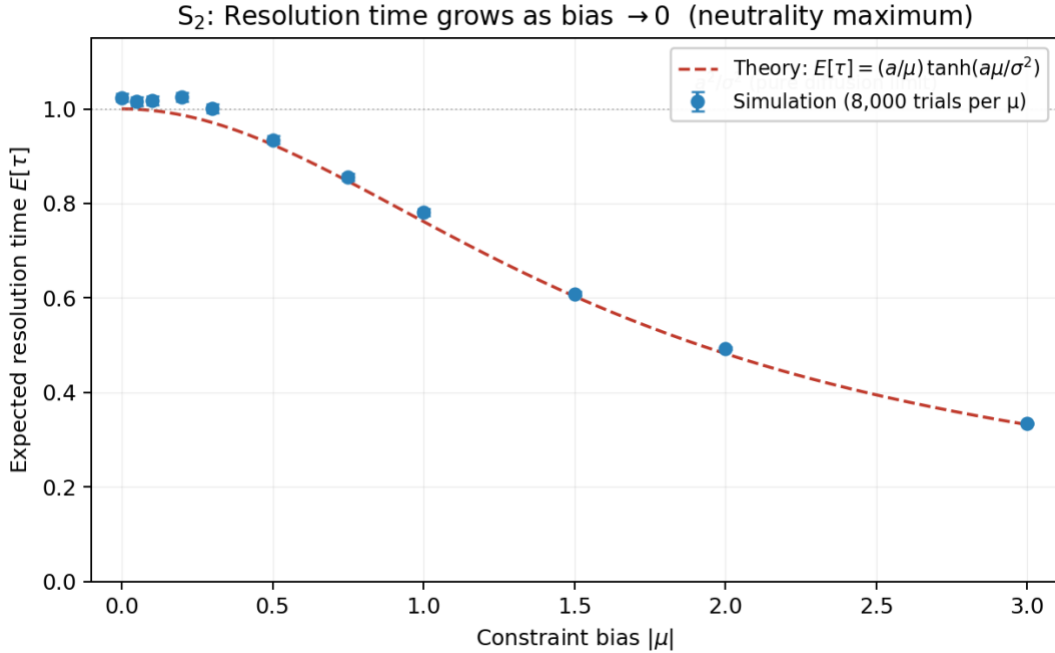


Figure 1. Expected resolution time for a one-dimensional drift-diffusion process with absorbing boundaries at $\pm a = \pm 1$, $\sigma = 1$, and $z_0 = 0$. Euler-discretized Monte Carlo estimates (8,000 trials per μ) approximate the theoretical curve $E[\tau] = (a/\mu) \tanh(a\mu/\sigma^2)$ (dashed); slight upward bias near $\mu = 0$ reflects finite-time-step boundary detection in the Euler simulation. At zero drift, expected resolution time attains the diffusion-only ceiling $a^2/\sigma^2 = 1$; it decreases monotonically as $|\mu|$ grows. Simulation code is in Appendix A.

5. Connection to UCT framework

In the Universal Collapse Theory kernel, realization is collapse under constraints: a constraint set K selects admissible outcomes from $\mathcal{Q}$ via a map C^K . When K weakly differentiates between admissible

outcomes, the collapse process samples the symmetric region for longer before committing. Theorem 1 makes this quantitative in the simplest formalism: drift-diffusion under absorbing boundaries.

This is *operational* S_2 : a measurable monotone latency response to constraint symmetry, under a stated formalism. It is not a claim that all decision latency reflects neutrality, that the formalism captures every domain, or that latency is the only signature of S_2 . The independence audit — whether ΔK is measured cleanly rather than inferred from latency itself — is what does the work in any real domain.

6. Limitations

The theorem is conditional on the drift-diffusion formalism with constant drift, constant noise, and fixed symmetric absorbing boundaries. Real systems may exhibit time-varying drift, multiplicative noise, asymmetric or moving boundaries, or higher-dimensional state spaces. The qualitative S_2 hypothesis may extend beyond this setting, but rigorous bounds in those settings require domain-specific analysis. The *formalism choice* itself is also a load-bearing decision in this note: drift-diffusion is the cleanest one-line lemma, but Wald sequential analysis, Kramers escape, and Markov spectral-gap bounds each give competing characterizations of S_2 . A future v1.x extension may either retain drift-diffusion as primary, with the others as bridging remarks, or formally state and compare bounds across all four formalisms.

7. Open extensions

Items reserved for future extension (paralleling the v1.0 $\rightarrow$ v1.x trajectory of TN- S_1):

- **Formalism choice.** Drift-diffusion is the primary formalism in this note; Wald, Kramers, and absorbing Markov are retained as analogies in §3, not parallel bounds. A future v1.x extension could present a comparison table of S_2 -style bounds across all four formalisms.
- **Theorem rigor.** The first-passage formula is derived self-contained from the Kolmogorov backward equation in §2, and monotonicity is proved via the sign of $h'(x)$ where $h(x) = \tanh(x)/x$. Karatzas & Shreve and Redner are retained as reference texts, not as load-bearing citations.
- **Reference list.** The reference list anchors first-passage theory (Redner), DDM in cognition (Ratcliff & McKoon), absorbing Markov chains (Kemeny & Snell), and metastability (Hänggi/Talkner/Borkovec); DOIs are included for all non-trivial references. Domain-specific sources for stochastic biology (Gillespie, Balázsi/van Oudenaarden/Collins) or genotype-phenotype neutral networks (Ahnert) remain candidates for a future v1.x version that gives those candidate-instance domains in §4 non-trivial weight.
- **Sample-complexity statement.** Theorem 1(c) gives μ_{ϵ} explicitly as $(\sigma^2/a) \cdot x_{\epsilon}$, with x_{ϵ} defined by an implicit transcendental equation, plus a small- ϵ asymptotic $\mu_{\epsilon} \approx (\sigma^3/a^2) \cdot \sqrt[3]{3\epsilon}$. Since $(\tanh x)/x$ has no elementary closed-form inverse, a future v1.x extension could provide a numerical recipe for x_{ϵ} or add a conservative analytic inequality for small ϵ .
- **Cross-citation.** A future T16 S_2 demonstration (e.g., COGITATE-style perceptual-resolution latency, developmental fate-timing study) cites this note as the formal lemma being tested.

Appendix A. Reproduction code

The following Python script reproduces the simulation and theoretical curve underlying Figure 1 (drift-diffusion first-passage simulation, $a = 1$, $\sigma = 1$, $dt = 5 \times 10^{-4}$, 8,000 trials per μ); minor visual styling may differ from the embedded manuscript figure. Dependencies: NumPy and Matplotlib only.

```
import numpy as np
import matplotlib.pyplot as plt

def simulate_fp(mu, sigma=1.0, a=1.0, dt=5e-4, max_t=20.0, n_trials=8000, seed=0):
```

```

rng = np.random.default_rng(seed)
n_steps = int(max_t / dt)
z = np.zeros(n_trials)
tau = np.full(n_trials, max_t)
resolved = np.zeros(n_trials, dtype=bool)
sqrt_dt = np.sqrt(dt)
for t in range(n_steps):
    if resolved.all(): break
    incs = mu * dt + sigma * sqrt_dt * rng.standard_normal(n_trials)
    z[~resolved] += incs[~resolved]
    newly = (np.abs(z) >= a) & (~resolved)
    tau[newly] = (t + 1) * dt
    resolved |= newly
return tau.mean()

def E_tau_theory(mu, sigma=1.0, a=1.0):
    if abs(mu) < 1e-9: return a*a / (sigma*sigma)
    return (a / mu) * np.tanh(a * mu / (sigma * sigma))

mus = np.array([0.0, 0.05, 0.1, 0.2, 0.3, 0.5, 0.75, 1.0, 1.5, 2.0, 3.0])
emp = [simulate_fp(m, seed=int(m*1000)+1) for m in mus]
mus_fine = np.linspace(0, 3, 200)
plt.plot(mus_fine, [E_tau_theory(m) for m in mus_fine], "--",
         label=r"Theory:  $E[\tau] = (a/\mu) \tanh(a\mu/\sigma^2)$ ")
plt.plot(np.abs(mus), emp, "o", label="Simulation (8,000 trials per  $\mu$ )")
plt.axhline(1.0, linestyle=":", linewidth=1)
plt.text(0.05, 1.025, r" $a^2/\sigma^2$  ( $\mu=0$  ceiling)")
plt.grid(True, alpha=0.25)
plt.xlabel(r"Constraint bias  $|\mu|$ "); plt.ylabel(r"Expected resolution time  $E[\tau]$ ")
plt.title(r" $S^2$ : Resolution time grows as bias  $\rightarrow 0$  (neutrality maximum)")
plt.legend(); plt.tight_layout(); plt.show()

```

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AI Disclosure

AI tools were used to assist with manuscript preparation. The underlying theory, arguments, and interpretive claims are the author's own, and the author takes full responsibility for the content.

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